Vector Algebra

Multiple Choice Question

Q: 1 $\vec{u}$, $\vec{v}$ and $\vec{w}$ are three non-zero vectors that are neither parallel nor perpendicular to each other.
For which of the following will the product DEFINITELY be a vector?

(i) $(\vec{u} \cdot \vec{v}) \times \vec{w}$
(ii) $(\vec{u} \times \vec{v}) \cdot \vec{w}$
(iii) $(\vec{u} \times \vec{v}) \times \vec{w}$

1 only (ii)
2 only (iii)
3 only (i) and (iii)
4 all - (i), (ii) and (iii)

Free Response Questions

Q: 2 $\vec{p}$ and $\vec{q}$ are two collinear vectors. **[1]**

State whether the statement below is true or false.
Give a valid reason for your answer.

$(\lambda\vec{p} + \beta\vec{q}) \times \vec{q} = 0$, where λ and β are scalars.

Q: 3 There are two vectors $\vec{u}$ and $\vec{v}$ such that $\vec{u} \cdot \vec{v} = 0$. **[1]**

Find the projection of the vector $\vec{u}$ on the vector $\vec{v}$.
Give a valid reason for your answer.

Q: 4 **State whether the following statement is true or false. Give a valid reason.** **[1]**

If the angle between $\vec{p}$ and $\vec{q}$ is obtuse, then $\vec{p} \cdot \vec{q} < 0$.

Q: 5 If $\vec{p}$ and $\vec{q}$ are unit vectors, show that the angle between $\vec{p}$ and $\vec{p} + \vec{q}$ is the same as the angle between $\vec{q}$ and $\vec{p} + \vec{q}$. **[1]**

Q: 6 The adjacent sides of a parallelogram, PQRS, are represented by $\vec{a}$ and $\vec{b}$. **[2]**

If $\vec{a} \cdot \vec{b} = 0$, what type of parallelogram is PQRS? Give a valid reason.

Q: 7 Show that any two non-zero vectors $\vec{u}$ and $\vec{v}$ are perpendicular if $\left|\vec{u} - \vec{v}\right| = \left|\vec{u} + \vec{v}\right|$. **[5]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3

Q.No	What to look for	Marks
2	Writes true.	0.5
	Gives a valid reason. For example, since $\vec{p}$ and $\vec{q}$ are collinear, $(\lambda\vec{p} + \beta\vec{q})$ is collinear to $\vec{p}$. This means that $(\lambda\vec{p} + \beta\vec{q})$ is parallel to $\vec{p}$. Hence, their cross product is zero.	0.5
3	**Writes that the projection vector will be a zero vector.**	0.5
	Gives the reason that the projection of vector $\vec{u}$ on vector $\vec{v}$ is given by $\frac{\vec{u}.\vec{v}}{\lvert\vec{v}\rvert}$ and since the dot product is 0, the projection vector is a zero vector.	0.5
4	**Writes true.**	0.5
	Justifies as follows: $\vec{p} \cdot \vec{q} = p \times q \times \cos\theta$, where $\cos\theta$ is negative when $90^\circ < \theta < 180^\circ$.	0.5
5	Writes that $\vec{p} + \vec{q}$ represents the third side of the triangle whose other two sides are $\vec{p}$ and $\vec{q}$ with magnitude 1 unit each(as they are unit vectors). (Award full marks if pictorial/vector explanation is provided.)	0.5
	Concludes that both $\vec{p}$ and $\vec{q}$ make equal angles with $\vec{p} + \vec{q}$ as they form an isosceles triangle.	0.5
6	Writes that as $\vec{a} \cdot \vec{b} = 0$, $\lvert\vec{a}\rvert \times \lvert\vec{b}\rvert \times \cos\theta = 0$.	0.5

Q.No	What to look for	Marks
	Reasons that $\|\vec{a}\| \neq 0$ and $\left\|\vec{b}\right\| \neq 0$ as they need to form a parallelogram.	0.5
	Deduces that cos θ = 0 and hence θ = 90°.	0.5
	Concludes that PQRS is a rectangle as the sides are intersecting at 90°.	0.5
7	Considers vectors $\vec{u}$ and $\vec{v}$ as: $\vec{u} = p\hat{i} + q\hat{j} + r\hat{k}$ $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$	0.5
	Uses the condition $\|\vec{u} - \vec{v}\| = \|\vec{u} + \vec{v}\|$ to write the following: $(p-x)^2 + (q-y)^2 + (r-z)^2 = (p+x)^2 + (q+y)^2 + (r+z)^2$	1.5
	Simplifies the above equation to obtain $px + qy + rz = 0$.	1.5
	Uses the above step to find the dot product of the two vectors $\vec{u}$ and $\vec{v}$ as: $\vec{u} \cdot \vec{v} = px + qy + rz = 0$	1
	Writes that the vectors are perpendicular since their dot product is equal to zero.	0.5